On the statistical analysis and probabilistic modeling of ship maneuvering results for waterway design[*]

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Abstract

A new approach has been developed to estimate the probabilities of ship's accident (collision or grounding) on basis of ship maneuvering results. Assuming that the trajectories of ship's track (cross or swept path tracks) are considered as the response ensemble of either a stationary or non-stationary random processes, the study then concentrates on defining the response characteristics by analyzing of their power spectrum density. Finally, the extreme statistics of ship exceeding a level of the channel limits are then determined on the basis of this information. In that way, it subsequently becomes possible to study channel width and channel depth in an integrated manner so that actual probabilities of ship grounding for the channel as whole will be obtained. Numerical examples have been given, the proposed approach can therefore be quantitatively evaluated.

KEYWORDS: STATIONARY AND NON-STATIONARY RANDOM PROCESSES, RESPONSE ASSEMBLE, SHIP'S GROUNDING, SHIP HANDLING SIMULATOR, DISTRIBUTIONS, SPECTRAL BANDWIDTH

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1 Introduction

Maritime simulation is a reliable and indispensable tool in the assessment of navigational safety of a ship in conjunction with harbors and fairways. The main application focuses essentially on channel design to indicate the ship's maneuverability and possible accident occurrence in relation to the human behavior and environmental conditions.

From the view point of port engineering, the analysis of the risk of accident in each zone of the approach channel on the basis of ship's track and swept path is extremely interesting.

The most common analysis methods are those that consider the different channel passage sections to be independent from each other. The analysis thus concentrates on defining the individual starboard and port side distribution functions. The probability of collision or grounding in each of the sections can then be computed by defining the lateral limits exceeding the navigable zone in the Gaussian distributions [4, 2] and [8]. The essential limitation of these methods lies in the fact that there is interdependency between transits in subsequent cross-sections of a channel. The analysis of the separate cross sections can only give separate estimates of the probability of exceeding the channel border in each particular cross section, it can never indicate for the channel as whole. A relatively simple way of determining the risk level of the entire channel is by dividing the channel into a few homogeneous parts then determining for each part the probability of the channel border being exceeded. The length of an independent part relates to a half wavelength of the ships track in the channel, which was estimated in [10] by several ship's length. Again, the critical point resides in the fact that the estimated wavelength of the ships tracks was defined simply by using judgment and expert rating.

The other method with a different focus was demonstrated by A. Burger [12]. The method has been developed by taking the interdependence of successive passage sections into account, then either using a linear regression model or Markov chains to determine the interdependency of transits in subsequent cross sections. However, this methodology still needs considerable effort to develop and it is also very costly because it requires a large number of the real time simulations [8]. For this reason, this method has not been applied in practical engineering yet.

The purpose of this paper is thus to develop a new approach to addressing of this problem. Another important matter concerning this study is establishing of real probabilistic model of the ship's grounding for design of waterways.

2 Methodologies

Stationary process of ship - pilot behavior

It is necessary to maintain the balance between the required competence by navigational environmental and the competence that a mariner can attain for safer ship's operations [5]. To ensure that the required navigational safety is achieved, the mariner must keep the ship stable so that the fluctuations during the ship's maneuvering process are minimized to maintain the track as close as possible to the desirable track throughout transit. The magnitude of these fluctuations vary depending, of course, not only on the different environmental conditions but also on the different competencies of mariners and even changing randomly for one mariner with different tests in the same condition. However the fluctuations, for a certain maneuvering condition and one mariner, will become "stable" or "stationary" after he has taken several tests. This means that for any mariner "non-stationary" or "unstable" process of ship's maneuver under his control should be avoided to facilitate the navigational safety. It is therefore possible that the ship's response (track, swept path, course, etc.) can therefore be viewed as the output signals of random stationary process and, as indicated above, as having the Gaussian distributions. It should be also remembered that it is quite applicable even more relevant to the vertical motions of ship.

The above statement has been verified using the technique, so called "Reserve Arrangement Test", as given in Appendix B. The data from many real time simulations were collected for the verification; one of which has been presented in Section 3 as a typical example.

Properties of a random Gaussian stationary process

In the engineering design the whole channel should be designed according to different parts so that the variations in local maneuvering conditions of each part during a transit can be neglected, and the technical specifications (width, depth, slope, ...) assigned along each part should be equal. It should be noted that the part lengths of channel, which may have several thousands meters, are quite different from the cross sections as discussed in Section 1, where the length interval is, let's say, $50m - 100m$ usually. Assume that a real time simulation is set up for a part of the channel with the length $L(m)$, a mariner completes the trail tests with the period $T(sec)$, and $x_i(t)$ is sample record (sample function) of the ship which is the ship's position recorded at predetermined time intervals during trial $i - th$. Let us assume that the ship exceeds the channel border rarely that successive up-crossings of a specified level are independent and can therefore be modeled as the Poisson process. Under these assumptions probability $P(b, T)$ that the response assemble $\{x(t)\}$ (the symbol $\{\}$ denotes an ensemble with the number of the sample records is n_s, we omitted n_s to simplify the notation) will cross at level $x = b$ (b is considered as a half of the channel width) at least once during a period $T(sec)$ given by [14]

$$P(b, T) = 1 - exp(-\nu_b T) \tag{1}$$

Where ν_b is the mean rate of crossing with level b, for the Gaussian response process, ν_b can be expressed as

$$\nu_b = \frac{1}{2\pi}\sqrt{\frac{m_{2x}}{m_{ox}}} exp\left[-\frac{1}{2}\frac{(b - \hat{\mu}_x)^2}{m_{ox}}\right] \tag{2}$$

Where m_{ox} and m_{2x} represent zero and second moments of the assemble $\{x(t)\}$, respectively and $\hat{\mu}_x$ represents the assemble mean value of $\{x(t)\}$, which can be determined by the following equations

$$\begin{aligned} \hat{\mu}_x &= \frac{1}{n_s}\sum_{i=1}^{n_s} x_i(t) && (3) \\ m_{ox} &= \int_{-\infty}^{\infty} \hat{S}_{xx}(\omega)\mathrm{d}\omega && (4) \\ m_{2x} &= \int_{-\infty}^{\infty} \omega^2 \hat{S}_{xx}(\omega)\mathrm{d}\omega && (5) \end{aligned}$$

Where n_s is the number of trials; $\hat{S}_{xx}(\omega)$ is the power spectral density(PSD) describes the distribution of the mean-square value of the assemble $\{x(t)\}$ over the frequency domain. Based upon the assemble $\{x(t)\}$, $\hat{S}_{xx}(\omega)$ can be determined by digital computer using fast Fourier transform algorithm. The major steps for this computation are emerged in the Appendix B.

Record length requirement

One of the most important features to this approach is to determine the total record length requirement, T_r, of the response assemble (i.e. the minimum total number of observations, N, in the assemble) to obtain a predetermined degree of accuracy of the power spectral estimate. It is somewhat similar to the required number of trails n_s per environmental condition, generally between 8 and 15 [8], in the other methods, the relationship between them can be clearly

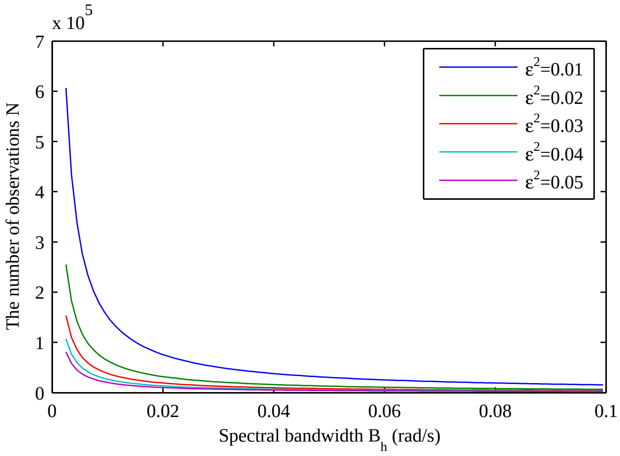

Figure 1: Relationship between N and B_h for various ϵ^2

expressed by

$$N = \frac{T_r}{\Delta t} = \frac{T}{\Delta t} n_s \tag{6}$$

Where Δt(sec) is the time interval for which the observations are recorded.

In practical, to measure a precision of the spectral estimate, its normalized random mean square error (rms) is widely used. The minimum total number of observations required for a specified rms error, ϵ, can be determined by using following equation [11]

$$\epsilon^2 = \frac{3.5044}{(NB_h)^{5/4}} \tag{7}$$

Where B_h is the spectral bandwidth (SB), it is measured [11] as the distance between the half-power points ω_1, ω_2 ($\omega_1 < \omega_0 < \omega_2$; ω_0 is the peak frequency) which are defined by $S_{xx}(\omega_1) = S_{xx}(\omega_2) = 0,5S_{xx}(\omega_0)$. Thus, $B_h = \omega_1 - \omega_2$. In case the PSD has a single peak at $\omega_0 = 0$, B_h is thus approximated to ω_2.

Eq. 7 implies that for a prescribed degree of the precision, N can be estimated as a function the spectral bandwidth B_h. Unfortunately, this is not always possible since B_h is usually unknown parameter prior to data collection. Certain assumptions based upon a prior knowledge of the spectral estimates and engineering judgement should therefore be required.

However, in preliminary stage (e.g. for setting up experiments), we are hardly ever interested in estimating just one value of N; in general we would wish to estimate N over a possible range of B_h (because data are usually limited and actual parameter of B_h is likely never known). Then we can select a value of B_h in lower bound and increase N for a higher precision. Concerning the study under discussion, it is highly probably supposed that the response assemble has commonly very long wavelength components; its spectral bandwidth is mostly lower than $0.1(rad/s)$. Fig. 1 shows the relationship between N and this range of B_h for various normalized rms error values. For example, in specified case, we wish to achieve $\epsilon^2 = 0.01$, using the prior information we may choose B_h, let's say between $0.005 - 0.01$. From Eq. 7, a value of N can be defined between 10000 and 15000 for the first trails.

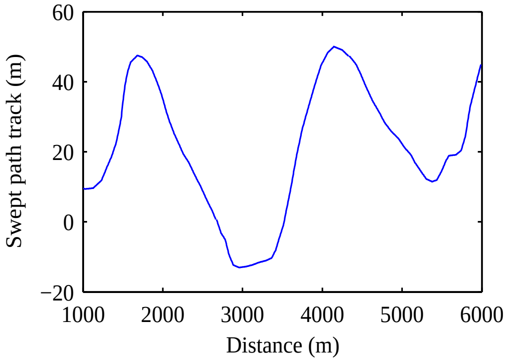

Figure 2: A sample record of ship's track (with additional post-processing)
(trial condition: wind $7Bft$ (SE), wave $H_s = 2m$ (SE) and current velocity $0.6m/s$ (SN))

3 Real time simulations

A real case is used for analysis and verification of the foregoing approach. The real time simulations were carried out at the Southern entrance channel to Ennore Coal Port, India [6]. The modeled channel with the length of $6km$ is aligned 345° North. The simulations were executed with the bulk carrier 65000DWT by four local pilots. A total of 37 trials with various environmental conditions were performed, in which 8 succeeded swept path tracks of the extreme conditions from km+1.000 to km+6.000 were taken for this study. Since the procedure of calculation on the starboard side is the same as those on the port side, only the latter is considered in the following. An example of a sample record of the swept path is described in Fig. 2.

Test for stationary

Data of the port swept paths from 8 trials are considered as a response assemble $\{x(t)\}$ (i.e. 8 sample records). For each having $N = 900$ data values recorded at time interval $\Delta t = 1s$ thus making the total time period of a transit $T = 900s$.

Divide the assemble record into 90 equal time intervals, the number of the observations in each interval therefore is $(900/90)8 = 80$. Compute a mean square value for each interval $(\bar{x}_1^2, \bar{x}_2^2, \ldots, \bar{x}_{90}^2)$ and align these values in time sequence. Count the number of times that $x_i > x_j$ for $i < j$ (each such inequality is called a reverse arrangement). From Eq. 21 in the Appendix A, the total number of reverse arrangement, denoted by A, is 1786.

Now let it be hypothesized that the assemble is stationary. From Table in the Appendix A, for $\alpha = 0.1, A_{90,1-\alpha/2} = A_{90,0.95} = 1766$ and $A_{90,\alpha/2} = A_{90,0.05} = 2238$. Hence the hypothesis is accepted at the level 1% of significance, since $A = 1786$ falls within the acceptance region between 1766 and 2238. Meaning that the response assemble is identified as being stationary.

Normalized rms error of the power spectral estimate

As revealed in the previous section, before the normalized rms error being able to be evaluated, the spectral bandwidth has been preliminarily defined. First, using the assemble $\{x(t)\}$, determine an estimate of $\hat{S}_{xx}(\omega)$, the result is as shown in Fig. 2. Then we try to estimate B_h based on the estimated bandwidth $\hat{S}_{xx}(\omega)$. As the definition of B_h, the value of B_h may be derived from Fig. 3 about 0,04. Hence, for $N = T.n_s = 7200$ and from Eq. 7, ϵ^2 will be 0.0378. Now, one may wish to gain ϵ^2 with a precision of 0.05 only, the value of N needed is about 5070 and the respective number of trials will be reduced to 6. It can be realized that the precision of this approach depends only on the number of the observations, whereas the number of trails

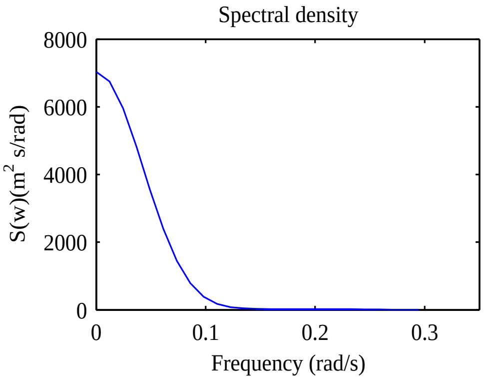

Figure 3: Power spectrum of the assemble

must be required large enough in the other methods.

Probabilities of ship's grounding

When the estimate of $\hat{S}_{xx}(\omega)$ is available, from Eqs in Section 2 the probabilities of ship's grounding can be quickly determined for a certain half of channel width. These probability results versus various half channel widths are plotted in the upper curve of Fig. 4.

As discussed above, if entire channel is viewed as consisting of a number of homogeneous parts, which relates to a half wavelength of the ship's swept path, the probability of ship's grounding can also be expressed by

$$P(b,T) = \left(\frac{L}{T}\right)\left(\frac{T_c}{2}\right)\int_b^\infty f(x)\mathrm{d}x \tag{8}$$

Where $f(x)$ is the density function of ship's position (swept path), which can, as indicated, be well described by Gaussian distribution; T_c is the mean zero-crossing period of the wavelength given as

$$T_c = 2\pi\sqrt{\frac{m_{ox}}{m_{2x}}} = 2\pi\sqrt{\frac{371.25}{2.34}} = 79.13s \tag{9}$$

The results for various half channel widths are also plotted in the lower curve of Fig. 4. It can be seen from this figure that these results become very slightly different from those computed by the proposed approach when b is greater that $110m$. The values of b within this area is very close to the designed point since an acceptable probability of ship's grounding is quite low, let us say about 3.10^{-5} [10].

4 Full probabilistic equation

Suppose that a ship be grounded when escaping from the channel border is not realistic for the case of a flooded channel or a dredged bank. It is only true if the channel bank is an upward rigid wall, in such a case that a ship appears to be collapsed rather than grounded. Actual probability that a ship is grounded, as illustrated in Fig. 5, is given by [3]

$$P_g = P[Z \cup (Y \cap X)] \tag{10}$$

$$P_g = P(Z) + P(X)P(Y|X) - P(Z)P(X)P(Y|X) \tag{11}$$

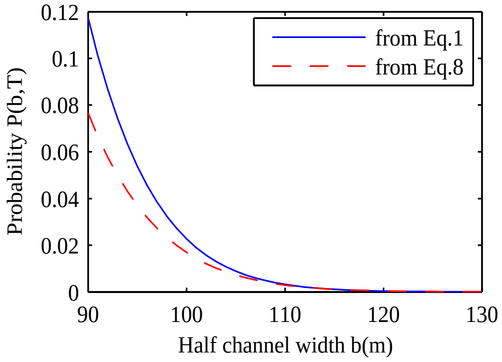

Figure 4: Probabilities of ship's excursion vs. half channel width

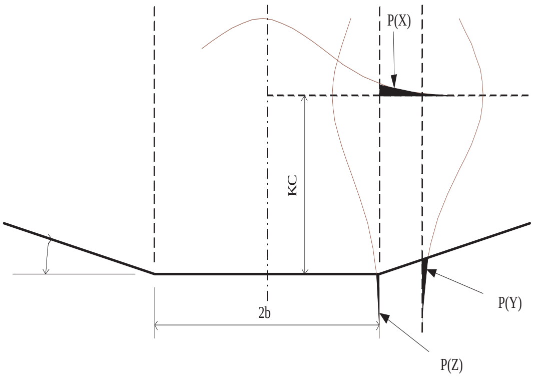

Figure 5: Grounding sceneries (black zones indicate probability of the grounding)

Where $P(Z)$ is the probability of ship's grounding inside the channel border; $P(Y)$ is the probability of ship's grounding during excursion from the channel border; $P(X)$ is the probability of ship's excursion from the channel border, as discussed above, $P(X) = P(b,T)$; $P(Y|X)$ conditional probability of ship's grounding during the excursion.

Consider a pair of the response assemble $\{x(t)\}$ and $\{y(t)\}$ which respectively represents the horizontal and vertical motions of ship. It is perhaps not surprising that $\{x(t)\}$ and $\{y(t)\}$ are completely uncorrelated as shown in Fig. 6, where $x(t)$ is a sample taken from the assemble $\{x(t)\}$; $y(t)$ is a sample of the vertical motion obtained in the same environmental condition. Eq. 11 can therefore be rewritten as

$$P_g = P(Z) + P(X)P(Y) - P(Z)P(X)P(Y) \tag{12}$$

Assume that the ship being progressed with the under-keel clearance KC (m); the channel has a type of flood bank with a slope angle α (degree) as shown in Fig. 5. It is as known that vertical motions of ship in waves can be considered as the Gaussian stationary process with the mean value $\mu_y = 0$. So Eq. 2 is applied to $\{y(t)\}$ as follows

$$\begin{aligned} \nu_h &= \frac{1}{2\pi}\sqrt{\frac{m_{2y}}{m_{oy}}} exp\left[-\frac{1}{2}\frac{h^2}{m_{oy}}\right] && (13) \\ h &= KC - [x(T_e) - b]tg\alpha && (14) \end{aligned}$$

Where $[x(T_e) - b]$ is considered as an exceeding distance (off-track) from the channel border during an excursion time period $T_e(sec)$, which varies from time to time for each occurrence

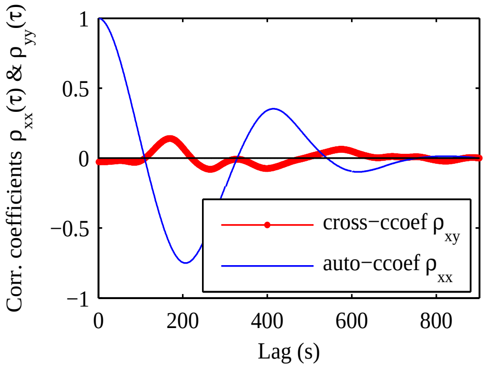

Figure 6: Cross- and auto- correlation coefficients

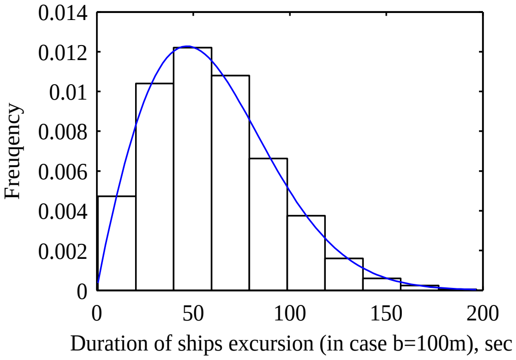

Figure 7: Probability density function of the time excursion fitted with Weilbul distribution

of ship's excursion. It should be realized that the further a ship moves a way from the center of the channel, the longer time which a mariner will need to steer the ship back to a desired track. An effort [3] has been made to investigate this phenomenon. In this case, the probability density function of the time excursion was found, as shown in Fig. 7 for the case of half channel width $b = 100m$, to fit well with the Weibull distribution.

Monte Carlo simulation

Using the Monte Carlo method, the probability of ship's grounding during excursion can be estimated by taking following steps:

- Generate a stochastic value of the time excursion as defined in Fig. 7;
- Determine the mean rate of crossing v_h as shown in Eq. 13;
- Determine $P(v_h, T_e)$ as defined similarly in Eq. 1;
- Repeat the above procedure n times, the probability of ship's grounding during a transit excursion can be estimated as

$$P(Y) = \sum_{i=1}^{n} P_i(\nu_h, T)/n \tag{15}$$

Numerical example

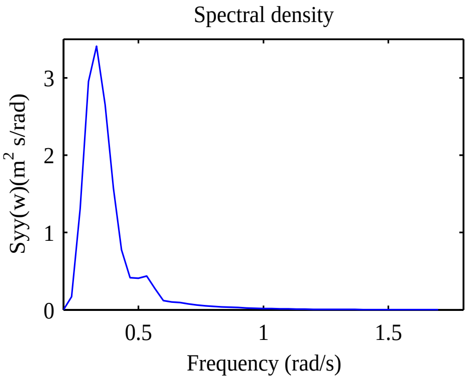

Figure 8: Spectral density of the motion at a critical point

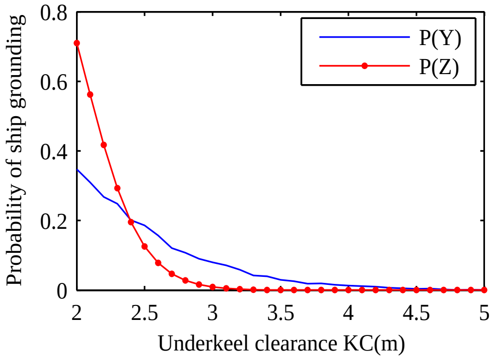

Figure 9: Probabilities of ship's grounding for varying KC

Continue the foregoing example by extending the condition that the spectrum of vertical motions of the ship, as shown in Fig. 8, has been derived from the numerical model [7] in the same wave condition as mentioned above.

The probabilities of ship's grounding versus different values of under-keel clearances KC for half channel width $b = 100m$ are plotted in Fig. 9. It is interesting to observe that probabilities of ship's grounding during excursion from the channel border, $P(Y)$, are even less than $P(Z)$ for the values of under-keel clearances KC are less than $2.3m$. The curve of $P(Z)$ is quickly dropped with KC increases; while $P(Y)$ is more slowly reduced. Finally, using Eq. 12 actual probabilities of ship grounding for the whole transit can be estimated with varying channel widths and under-keel clearances.

5 Non-stationary process of ship's response

The response process $\{x(t)\}$ may be non- stationary in such a special case that meteorological and hydrodynamic conditions vary so frequently and considerably along approach channel that the separation of the channel into some parts is no longer useful because a part length of the channel will be too short in comparison with the wavelength of the ship's track. However, it is highly likely that the response process $\{x(t)\}$ may become stationary when mariner is getting adapted to such condition. Much effort should be made to investigate the characteristics of ship's response in this particular case.

Wigner distribution

Similarly to the analysis for response process $\{x(t)\}$ is stationarity, the core of this approach is to determine the power spectrum for non-stationary process of the sample records $\{x(t)\}$ by using Wigner distribution (WD). The analysis procedures of the WD for non-stationary processes have essentially the same type of physical interpretation as the spectra of the stationary processes, the main distinction being that whereas the spectra of a stationary process describes the power-frequency distribution for the whole process (i.e. over all time), the WD is time dependent and describes the local power-frequency distribution at each instant time [13].

The WD of a real signal $x(t)$ is given by [9].

$$W(t,f) = \int_{-\infty}^{\infty} z(t+\tau/2)z^*(t-\tau/2)e^{j2\pi f\tau}\mathrm{d}\tau \tag{16}$$

Where $z(t)$ is the analytic signal associated with $x(t)$, $z^*(t)$ represents the complex conjugate of $z(t)$, which are defined in the Appendix C.

Integration of Eq. 16 over $(-\infty,\infty)$, the time - frequency spectrum of the WD at every time t and frequency f will be defined. Thus, so on, integration of $W(t,f)$ over all f gives the instantaneous signal power of $\{x(t)\}$ at time t. Also integration of $W(t,f)$ over all t gives the power spectral density function of $\{x(t)\}$ at frequency f. The total energy in $\{x(t)\}$ over the whole (f,t) plane is given by

$$\iint_{-\infty}^{\infty} W(f,t)\mathrm{d}f\mathrm{d}t = \iint_{-\infty}^{\infty} W(f,t)\mathrm{d}t\mathrm{d}f \tag{17}$$

Probability of ship's excursion

The response process $\{x(t)\}$ is a non-stationary when viewed as a whole. However, in the (f,t) plane it may be possible to separate the non-stationary power spectral function into piecewise stationary segments even the environmental conditions can be sequentially changed . The location and the number of segments for separation can be found by detecting the WD. This means that any definition of the spectrum of the WD for a non-stationary process should reduce to the classical definition when, in particular, the process is stationary. Eqs. 1 and 2 will therefore be applicable to the present context. It is assumed that, of course accepting some minor errors, the segments are independent each other, the total probability of ship's excursion will be determined as

$$P(X) = 1 - \prod_{i=1}^{n} P_i(X) \tag{18}$$

Where $P_i(X)$ is probability of ship's excursion in the segment ith, n is the number of segments.

6 Conclusions

Since the whole channel can be designed in an integrated manner using this probabilistic model, an optimal design can be carried out by balancing between channel depth and channel width, a total dredging volume will be then minimized for a specified acceptable risk of ship's accident. Moreover, actual probability of ship's grounding will appropriately estimated, which is the most important factor to establish safety criteria for the design and navigational operation of waterways as well.

Probability that a ship is really grounded during excursion from the channel limits, as indicated by P(Y) in Fig. 9, is totally different from probability of ship's excursion, as given by P(X) in Fig. 4. The difference is subject to the magnitude of minimum under-keel clearance KC.

Application of this approach to particular cases (e.g. very long channel but stationary navigational condition) may get some benefits because the number of trails is less than that required in the other methods.

7 Future works

Before performing the fast Fourier transform, it is necessary to preprocess data by applying an appropriate "time window" to avoid "leakage" and "amplitude ambiguity" [1]. There are numerous such windows in current use. Further investigation should be carried out to define an appropriate one for applying to this situation.

Emphasis should be placed on the sensitive analysis between the number of the observations and the normalized rms error as well as the precision of the probability results.

The spectral analysis approach as well as the WD technique, in particular, is a powerful tool in determining the time-frequency characteristics of stationary or highly non-stationary signals and applied widely not only in the engineering design but also in the analysis of economic and social activities. However, characteristics of the ship's response signal process are different from all others (e.g. its power spectrum density has very narrow bandwidth in very low frequency range and it decays rapidly in higher frequency). Effort should therefore be devoted to make the method more appropriate in application to waterway design.

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Appendix A: The reverse arrangement test for stationary

Let it be hypothesized that the sequence of sample mean square values $(\bar{x}_1^2, \bar{x}_2^2, \ldots, \bar{x}_N^2)$ represents independent sample measurements of a stationary random variable with mean square value of ψ_x^2. If this hypothesis is true, the variations in the sequence of sample values will be random and display no trend, the hypothesis of stationarity is accepted. Otherwise, this is rejected. Meaning that any non-stationarity of interest will be revealed by trends in the mean square value of the data. For detecting trends in the sequence, the reverse arrangement test (RAT) is the most effective and powerful tool. The RAT is applied as a test for stationarity as follows [13]

Divide the assemble record into N equal time intervals where the data in each interval may be considered independent.

Compute a mean square value for each interval and align these values in time sequence.

Count the number of times that $x_i > x_j$ for $i < j$, each such inequality is called a reverse arrangement. The total number of reverse arrangement, denoted by A, is determined as follows.

$$h_{ij} = \begin{cases} 1 & \text{if } x_i > x_j \\ 0 & \text{otherwise,} \quad \text{then} \end{cases} \tag{19}$$

$$A = \sum_{i=1}^{N-1} A_i \qquad \text{where} \tag{20}$$

$$A_i = \sum_{j=1}^{N} h_{ij} \tag{21}$$

Since the sequence of N values are independent, then the total number of reverse arrangements is a random variable, with a mean and variance values as

$$\mu_A = N(N-1)/4 \tag{22}$$

$$\sigma_A^2 = (2N^3 + 3N^2 - 5N)/72 \tag{23}$$

Tabulation of percentage points for the distribution function of A presented in the following

Table 1: Percentage points of reverse arrangement distribution

N	α					
	0.990	0.975	0.950	0.050	0.025	0.010
40	290	305	319	460	474	489
50	473	495	514	710	729	751
60	702	731	756	1013	1038	1067
70	977	1014	1045	1369	1400	1437
80	1299	1344	1382	1777	1815	1860
90	1668	1721	1766	2238	2283	2236

Now, let it be hypothesized that the sequence of N values belong to stationary random process, where there is no trend. The acceptable region for this hypothesis is

$$[A_{N,1-\alpha/2} < A \leq A_{N,\alpha/2}] \tag{24}$$

where α is the level of significant of the test; and the value of $A_{N,\alpha/2}$ such that Prob$[A_N > A_{N,\alpha/2} = \alpha/2]$.

Appendix B: Estimation of power spectral density

In the problem under discussion, because the response process x(t) is not egordic, PSD estimate $\hat{S}_{xx}(\omega)$ must be estimated from the response assemble $\{x(t)\}$.

Let each sample record $x_i(t)$ be represented by N data values, which recorded during the simulation with the time interval Δt, $\{x_{in}\}$, $n = 0, 1, \ldots, N-1$, $i = 1, 2, \ldots, n_s$.

The discrete frequencies for the finite Fourier transform process are given at

$$\omega_k = \frac{k}{T} = \frac{k}{N\Delta t}, \quad k = 0, 1, \ldots, N-1 \tag{25}$$

The Fourier components for each frequency are then given by

$$X_i(\omega_k) = \Delta t X_{ik} = \Delta t \sum_{n=0}^{N-1} x_{in} exp(-\frac{j2\pi kn}{N}) \tag{26}$$

Finally, the power spectral density function is estimated as

$$S_{xx}(\omega_k) = \frac{1}{n_s N\Delta t} \sum_{i=1}^{n_s} |X_i(\omega_k)|^2, \quad k = 0, 1, \ldots, N-1 \tag{27}$$

Appendix C: Estimation of the analytic signal, $z(t)$, for WD

The analytic signal, $z(t)$, is defined by [9]

$$z(t) = x(t) + j\tilde{x}(t) \tag{28}$$

Where $\tilde{x}(t)$ is the Hilbert transform of $x(t)$. Once can also write $z(t)$ in another form as

$$z(t) = A(t)e^{j\theta(t)} \tag{29}$$

Where $A(t)$ is called the envelop signal of $x(t)$ and $\theta(t)$ is called instantaneous phase signal of $x(t)$, which are expressed as [13]

$$A(t) = \left[x^2(t) + \tilde{x}^2(t)\right]^{1/2} \tag{30}$$

$$\theta(t) = tan^{-1}\left[\frac{\tilde{x}^2(t)}{x(t)}\right] = 2\pi f_0 t \tag{31}$$

The instantaneous frequency f_o is given by

$$f_0 = \left(\frac{1}{2\pi}\right)\frac{\mathrm{d}\theta(t)}{\mathrm{d}t} \tag{32}$$

It is a very simple transform to obtain $Z(f)$ from $X(f)$. One should compute $X(f)$ for all f and then define $Z(f)$ by $Z(0) = X(0)$ and

$$Z(f) = \begin{cases} 2X(f) & \text{for } f > 0 \\ 0 & \text{for } f < 0 \end{cases} \tag{33}$$

The inverse Fourier transform of $Z(f)$ give $z(t)$ with

$$\tilde{x}(t) = \mathrm{Im}[z(t)] \quad \text{and} \quad x(t) = \mathrm{Re}[z(t)] \tag{34}$$

For digital computations of the response assemble $\{x_i(n\Delta t)\}$ under study, here $n = 0, 1, \ldots, N-1$; $i = 1, 2, \ldots, ns$, Eq. 34 can be rewritten as

$$\begin{aligned} x(n\Delta t) &= 2\Delta f \mathrm{Re}\left[\sum_{k=0}^{N/2} X_i(k\Delta f) exp\left(\frac{j2\pi kn}{N}\right)\right] & (35) \\ \tilde{x}(n\Delta t) &= 2\Delta f \mathrm{Im}\left[\sum_{k=0}^{N/2} X_i(k\Delta f) exp\left(\frac{j2\pi kn}{N}\right)\right] & (36) \end{aligned}$$

Here, the factor $\Delta f = 1/(N\Delta t)$ with

$$X_i(k\Delta t) = \Delta t \sum_{n=0}^{N-1} x_i(n\Delta t) exp\left(\frac{-j2\pi kn}{N}\right) \tag{37}$$

Note that the values of $X_i(k\Delta t)$ are needed only from $k = 0$ up to $k = N/2$, where $Nyquist$ [1] frequency occurs, to obtain the digitized values of $x(n\Delta t)$ and its the Hilbert transform $\tilde{x}(n\Delta t)$. The envelop signal of $x(n\Delta t)$ in Eq. 31 in discrete form is

$$A(n\Delta t) = \left[x^2(n\Delta t) + \tilde{x}^2(n\Delta t)\right]^{1/2} \tag{38}$$

The analytic signal $z(t)$ is then estimated from Eq. 28